

# The ground states of the two-component order parameter superconductor

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**Abstract.** - We show that in presence of an applied external field the two-component order parameter superconductor falls in two categories of ground states, namely, in the traditional Abrikosov ground state or in a new ground state fitted to describe a superconducting layer with texture, that is, patched regions separated by a phase difference of  $\pi$ . The existence of these two kinds of ground states follows from the sole assumption that the total supercurrent is the sum of the two individual supercurrents and is independent of any consideration about the free energy expansion. Uniquely defined relations between the current density and the superfluid density hold for these two ground states, which also determine the magnetization in terms of average values of the order parameters. Because these ground state conditions are also Bogomolny equations we construct the free energy for the two-component superconductor which admits the Bogomolny solution at a special coupling value.

**Introduction.** – Many thermodynamic and transport phenomena in superconductivity can be explained on the basis of a macroscopic order parameter, as shown by the phenomenological approach of Ginzburg and Landau (GL), proposed many year before Leon Cooper found that a weak attraction binds electrons near to the Fermi surface to form pairs. From the GL theory A. A. Abrikosov predicted the existence of two types of superconductors according to the dimensionless coupling  $\kappa$ , thus proving that this theory is well suited to describe alloys, which are  $\kappa > 1/\sqrt{2}$  superconductors, able to sustain a stable vortex state in presence of an external applied field [1]. Central to the present analysis is the observation that at the core of Abrikosov's treatment lives a condition which is totally independent of the Ginzburg-Landau free energy. This condition is the requirement that the macroscopic order parameter fulfils a ground state equation and from it the supercurrent and the superfluid densities are shown to be directly related to each other. It also follows from this condition that the magnetization is proportional to the spatial average of the superfluid density, a mean field result that remains valid in the presence of thermal fluctua-

tions [2]. Therefore the ground state condition for the one-component order parameter (1COP) superconductor lives at a more fundamental level than the Ginzburg-Landau free energy expansion. To attain full predictive power from the magnetization expression the superfluid density must be determined in terms of fundamental parameters, and, at this point, the free energy considerations becomes valuable, as shown by Abrikosov, who obtained from the GL theory the magnetization as a function of the temperature  $T$  and of the external applied magnetic field,  $H$ , taken to be near to the upper critical field  $H_{c2}$  [3].

In this letter we show that for the two-component order parameter (2COP) superconductor the requirement that the macroscopic order parameter fulfils a ground state equation leads to just two possibilities, namely, the Abrikosov ground state, applied individually to each one of the components, and a new ground state that describes a textured superconducting layer because of the intrinsic phase difference of  $\pi$  between its distinct regions. Therefore we find here that for the 2COP superconductor the ground state condition also exists and lives in a more fundamental level than the free energy expansion. In case

the local field is equal to the applied one, the Abrikosov ground state locks the 1COP into the lowest Landau level, whereas this new 2COP ground state takes contributions from all Landau levels.

In 1976 E. Bogomolny [4–7] found, while working in string theory, extra properties for the GL theory valid for  $\kappa = 1/\sqrt{2}$ : first order equations, instead of the second order variational equations (Ampère’s law and the GL equation) solve the GL theory. The Bogomolny’s first order equations share a common feature with the Abrikosov’s treatment of the GL theory: one of these equations is exactly the Abrikosov ground state condition.

We also study here a 2COP free energy because the new ground state condition is one of its Bogomolny equations. This means that at a particular coupling, defined to be  $\kappa = 1/\sqrt{2}$ , like for the 1COP case, the minimum of this 2COP free energy is reached by first order equations instead of second order ones, which are then defined as 2COP Bogomolny equations. Therefore this 2COP free energy theory is a unique generalization of the 1COP GL theory. Previously proposed 2COP GL theories [8–12] do not contain this feature, namely, none of them is minimized by the present Bogomolny equations associated to this new ground state. We stress that the study of this 2COP free energy is just complementary to our major claim of a ground state condition independent of a free energy proposal.

Nowadays a broad range of experimental data is pointing towards common features for the superconductors whose pairing originates in two-dimensional layers, such as the existence of two fundamental energy scales [13, 14] and the competition of the superconducting state with a magnetic collective state [15]. This leads to akin properties for the layered superconductors albeit their distinct atomic structure. The most studied materials are the cuprates whose two-dimensional layers are made of copper and oxygen atoms [16–18]. Recently the pnictides brought excitement into the field because superconductivity originates in two-dimensional layers made of iron and arsenic atoms instead [19–21]. The origin of superconductivity, the microscopic nature of the pairing mechanism [27, 28], and the intrinsic magnetism are under current investigation [13, 16–21]. We propose here that the new 2COP ground state will help to understand the macroscopic properties of the cuprates and of the pnictides because of its intrinsic layered structure.

The new ground state is described by a 2COP,  $\Psi \equiv (|\psi_1| \exp(i\theta_1), |\psi_2| \exp(i\theta_2))$ , whose response to the presence of an applied field  $\vec{H} = H\hat{x}_3$  is a local magnetization proportional to  $\Psi^\dagger \vec{\sigma} \Psi$ . In components this magnetization is given by,

$$M_1 = -(\hbar q/mc) \langle \cos \theta |\psi_1| |\psi_2| \rangle, \quad (1)$$

$$M_2 = (\hbar q/mc) \langle \sin \theta |\psi_1| |\psi_2| \rangle, \text{ and,} \quad (2)$$

$$M_3 = -(\hbar q/2mc) \langle |\psi_1|^2 - |\psi_2|^2 \rangle, \quad (3)$$

where  $\theta = \theta_1 - \theta_2$  is the phase difference between the two

order parameters and  $\langle \dots \rangle$  means spatial average. Therefore there is a *local* transverse magnetization to the applied field, but the total average magnetization should vanish,  $M_1 = M_2 = 0$ , to avoid a spontaneous torque in the system. This is a novel feature, not present in the Abrikosov ground state, whose magnetization is fully oriented along the applied field. The key ingredient to make the *average* in plane magnetization to vanish is to introduce texture, which means that the *local* transverse magnetization flips sign by the phase shift  $\theta \rightarrow \theta + \pi$ . Phase texture in two-gap superconductors has been discussed in the literature [22]. It introduces interesting features such as the onset of solitons [23, 24] and an intercomponent Josephson interaction [25] that may directly relate to an abnormal AC loss peak [26].

The 1COP GL theory has been applied to the cuprates either by including a mass anisotropy or by the Lawrence-Doniach model, which treats superconducting layers coupled by the Josephson effect [1]. The present 2COP free energy allows for such generalizations that will be considered elsewhere.

**One-component order parameter (1COP) ground state.** – On the basis that the superconducting state can be described by a macroscopic wavefunction,  $\psi$ , the supercurrent density is  $\vec{J} = (q/2m) (\psi^* \vec{D} \psi + c.c.)$ ,  $\vec{D} = (\hbar/i) \vec{\nabla} - (q/c) \vec{A}$ . Without invoking the GL theory, Ampère’s law,  $\vec{\nabla} \times \vec{h} = (4\pi/c) \vec{J}$ , is exactly solved for the local field,  $\vec{h} = \vec{\nabla} \times \vec{A}$ , in terms of  $\psi$  by assuming the ground state condition,

$$D_+ \psi = 0, \quad (4)$$

where  $D_\pm = D_1 \pm iD_2$ . This ground state condition applies for a bulk superconductor with continuous axial symmetry along the applied field direction ( $\vec{H} = H\hat{x}_3$ ), such that fields only dependent on the coordinates orthogonal to it,  $(x_1, x_2)$ . From the real and imaginary parts of Eq.(4), we find that the supercurrent and superfluid densities are related by,

$$\vec{J} = -\frac{\hbar q}{2m} \vec{\nabla} \times (|\psi|^2 \hat{x}_3) \quad (5)$$

Integration of Ampère’s law gives that the local field is,

$$h_3 = H - \frac{\hbar q}{mc} |\psi|^2, \text{ and } 4\pi M = -\frac{\hbar q}{mc} \langle |\psi|^2 \rangle. \quad (6)$$

is the magnetization. For the 1COP case the spatial average value is over the orthogonal coordinates, such as for the magnetic induction,  $\vec{B} = \vec{H} + 4\pi \vec{M}$ , which becomes  $\vec{B} \equiv \langle h_3 \rangle \hat{x}_3 = (\int d^2x h_3/S) \hat{x}_3$ , since along the third direction and under translational invariance, it suffices to consider integration over the unit cell area  $S$ . For a constant field ( $A_1 = -Hx_2$ ,  $A_2 = 0$ ), Eq.(4) is just the lowest Landau level condition whose solution is

$$\psi = \sum_k c_k e^{ikx_1 - \frac{qH}{2\hbar c} (x_2 + \frac{\hbar ck}{qH})^2}. \quad (7)$$

The set of wavenumbers  $k$  and the constants  $c_k$  are determined by imposing periodic conditions to the order parameter and fixing the number of vortices within the unit cell area. One obtains that  $B = N\Phi_0/S$ , for  $N$  vortices within the cell. Notice that the order parameter of Eq.(7) together with the local magnetic field of Eq.(6) stem just from the ground state condition. Nevertheless they describe the vortex state without invoking the free energy, that only enters to select the vortex state of lowest free energy. As shown by E.H. Brandt [1], the order parameter of Eq.(7) and the local field of Eq.(6) provide an excellent description of the full GL free energy solution for fields in the range  $0.5H_{c2} \leq H \leq H_{c2}$ . Therefore the ground state condition of Eq.(4) does describe the GL theory for an applied field not necessarily near to  $H_{c2}$ .

**Two-components order parameter (2COP) ground state.** – Along the crystal's major axis, where the mass tensor is diagonal, and an appropriate combination of the two order parameters is taken, the supercurrent is expressed by  $\vec{J} = \vec{J}_1 + \vec{J}_2$ ,  $J_j = (q/2m) (\psi_j^* \vec{D} \psi_j + c.c)$ ,  $j = 1, 2$ . The Abrikosov ground state assumes translational invariance along the applied field such that the condition of Eq.(4) applies to both components, and the local field becomes  $h_3 = H - (hq/mc)(|\psi_1|^2 + |\psi_2|^2)$ . This new ground state satisfies the condition,

$$\vec{\sigma} \cdot \vec{D} \Psi = 0, \text{ where } \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad (8)$$

where  $\vec{\sigma}$  are the Pauli matrices, the generators of the SU(2) group:  $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ,  $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ , and  $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ . These matrices live on a vector representation of the O(3) group, and so this ground state contains an orientation and a direction in space, associated to the physical superconducting layer linked to the crystal's major axes. Take the two components of Eq.(8),  $D_3\psi_1 + D_-\psi_2 = 0$  and  $D_+\psi_1 - D_3\psi_2 = 0$  to separate the two order parameters in second order equations:  $[(D_1^2 + D_2^2) + D_3^2 - (q\hbar/c)h_3]\psi_1 = 0$  and  $[(D_1^2 + D_2^2) + D_3^2 + (q\hbar/c)h_3]\psi_2 = 0$ . We show that this new 2COP ground state is truly three-dimensional, although it describes fields that evanesce from a layer, opposite to the Abrikosov ground state, which is two-dimensional and describes a translational invariant bulk state along the third direction. We obtain the exponential decay away from the layer in presence of  $H$  such that the local field is dominated by the applied field,  $h_3 \approx H > 0$ . The kinetic energy within the orthogonal plane is positive,  $(D_1^2 + D_2^2)\psi_i \geq 0$ ,  $i = 1, 2$ , and so, we must have that  $D_3^2\psi_2 \leq 0$  in order to obtain a non-trivial solution for the second equation. We take the gauge  $A_3 = 0$  and that the layer is at  $x_3 = 0$ , such that  $\psi_i = \exp(-|q_3|x_3)\psi_i(x_1, x_2)$ ,  $i = 1, 2$ , to obtain that,

$$(D_1^2 + D_2^2) \psi_1 = [(\hbar q_3)^2 + \frac{\hbar c}{q} h_3] \psi_1 \quad (9)$$

$$(D_1^2 + D_2^2) \psi_2 = [(\hbar q_3)^2 - \frac{\hbar c}{q} h_3] \psi_2, \quad (10)$$

For the case  $h_3 = H$  the system admits the general solution,

$$\Psi = \sum_{n=0}^{\infty} C_n e^{-q_3(n)|x_3|} \begin{pmatrix} \psi_{(n)}(x_1, x_2) \\ \psi_{(n-1)}(x_1, x_2) \end{pmatrix}, \quad (11)$$

$$q_3(n) = \sqrt{2n \frac{qH}{\hbar c}}, \quad \psi_{(n-1)} = -\frac{i}{\hbar q_3(n)} D_+ \psi_{(n)}, \quad (12)$$

where  $(D_1^2 + D_2^2)\psi_{(n)} = (\hbar qH/c)(2n+1)\psi_{(n)}$  and  $\psi_{(-1)} = 0$  by definition. The first term retrieves the lowest Landau level condition for the 1COP ( $q_3 = 0$ ,  $\psi_2 = 0$  and  $D_+\psi_1 = 0$ ) and so the above 2COP contains both two and three dimensional contributions. We stress that each pair  $(\psi_{(n)}, \psi_{(n-1)})$  is bound by the same coefficients  $c_k$  and wave numbers  $k$ , but not necessarily two different pairs.

The major result of this letter is that under the ground state condition of Eq.(8), the supercurrent becomes

$$\vec{J} = -\frac{\hbar q}{2m} \vec{\nabla} \times (\Psi^\dagger \vec{\sigma} \Psi). \quad (13)$$

Integration of Ampère's law gives that,

$$\vec{h} = \vec{H} - \frac{\hbar q}{mc} \Psi^\dagger \vec{\sigma} \Psi, \text{ and } 4\pi \vec{M} = -\frac{\hbar q}{mc} \langle \Psi^\dagger \vec{\sigma} \Psi \rangle, \quad (14)$$

for the local magnetic field and for the magnetization, respectively. The latter is a three-dimensional average value defined as  $\vec{B} \equiv \langle \vec{h} \rangle = \int d^3x \vec{h}/V$  for the magnetic induction, where  $V$  is the volume of the unit cell.

In conclusion several ground states with elaborate magnetic patterns can be constructed from our explicit 2COP solution  $\Psi$ , given by Eqs.(11) and (12). As previously discussed these ground states are textured which means that they have neighbor regions phase separated by  $\pi$  in order to render that  $\langle \Psi^\dagger \sigma_1 \Psi \rangle = \langle \Psi^\dagger \sigma_2 \Psi \rangle = 0$ , yielding a vanishing magnetization along the layer. This in plane magnetization must vanish inside the bulk of the superconductor, and there are several ways to do it, either by considering two distinct layers with opposite in plane magnetization, stripes [27] or checkerboard patterns [29] inside a single layer.

**Virial relation.** – We provide another derivation of Eqs.(6) and (14) from the ground state conditions, Eqs.(4) and (8). This derivation brings insight into the kinetic energy and is based on the scalar virial relation [30–32] instead of the supercurrent density. The scalar virial relation holds for both the 1COP and the 2COP and states that the applied field times the magnetic induction is proportional to the average kinetic energy plus twice the average field energy:

$$\frac{\vec{H} \cdot \vec{B}}{4\pi} = \left\langle \frac{|\vec{D}\Psi|^2}{2m} \right\rangle + 2 \left\langle \frac{\vec{h}^2}{8\pi} \right\rangle, \quad (15)$$

The kinetic energy is expressed along the crystal's major axis, taken appropriate combinations of the two components, that bring it to the above form. We briefly review the 1COP case, considered by U. Klein and B. Pöttinger [33] sometime ago. The virial relation is  $HB/4\pi = \langle |D_1\psi|^2 + |D_2\psi|^2 \rangle / 2m + \langle h_3^2 \rangle / 4\pi$ , and from it, we obtain Eq.(6). To show this, express the kinetic energy in terms of the ground state condition of Eq.(4):  $\sum_{j=1}^2 |D_j\psi|^2 = \sum_{j=1}^2 \nabla_j (i\psi^* D_j\psi) + |D_+\psi|^2 + (\hbar q/c)h_3|\psi|^2$ . The total derivative term does not contribute to the average kinetic energy because of periodic boundary conditions. One obtains that  $H\langle h_3 \rangle / 4\pi = (\hbar q/2mc)\langle h_3|\psi|^2 \rangle + \langle h_3^2 \rangle / 4\pi$ . Then one easily verifies that the solution for this relation is indeed given by Eq.(6). Similarly the average kinetic energy of Eq.(15) can be expressed through the ground state condition, using that,

$$\langle |\vec{D}\Psi|^2 \rangle = \langle |\vec{\sigma} \cdot \vec{D}\Psi|^2 \rangle + (\hbar q/c) \langle \vec{h} \cdot \Psi^\dagger \vec{\sigma} \Psi \rangle, \quad (16)$$

The virial relation of Eq.(15) leads to the local field of Eq.(14), by introducing Eq.(8) into the above expression.

The kinetic energy density is intrinsically positive, and so, helpful to understand why in presence of  $H$  the 2COP has the top component in a Landau level higher than the bottom one, according to Eq.(11). Take a constant applied field along the third direction in Eq.(8), use Eqs.(14) and (16), and neglect the fourth order term under the assumption of weak order parameter because of the proximity to the normal state. It follows that  $\langle |\vec{D}\Psi|^2 \rangle / (\hbar q/c) \approx H\langle |\psi_1|^2 - |\psi_2|^2 \rangle$ , indicating that  $\langle |\psi_1|^2 \rangle \geq \langle |\psi_2|^2 \rangle$ . An interesting property is that  $\langle \Psi^\dagger \sigma_3 \Psi \rangle = 0$  holds for each pair  $\Psi$  given by  $(\psi_{(n)}, D_+\psi_{(n)}/i\hbar q_3(n))$ .

**Ginzburg-Landau Theory.** – The 2COP GL theory with Bogomolny solution has three temperature independent parameters,  $\alpha_i(T)$ ,  $i = 1, 2, 3$ , that form a vector and consequently introduce intrinsically preferred directions for the superconductor. Nevertheless it has a single dimensionless coupling that for  $\kappa = 1/\sqrt{2}$  admits a Bogomolny solution, as shown below.

We briefly review the derivation of the Bogomolny solution for the 1COP GL theory, whose free energy is  $F(\vec{B}, T) = \langle |\vec{D}\psi|^2 / 2m - \alpha(T)|\psi|^2 + \beta|\psi|^4 / 2 + \vec{h}^2 / 8\pi \rangle$ , where  $\alpha(T) = (T_c - T)$  near to the transition and  $\beta > 0$ , rendering possible to have a non-trivial solution  $\psi$  in case that  $\alpha(T) > 0$ . Expressing quantities in dimensionless units based on the coherence length,  $\xi(T)^2 = \hbar^2 / 2m|\alpha(T)|$ , and the thermodynamic magnetic field,  $H_c(T)^2 / 4\pi = \alpha(T)^2 / \beta$ , one finds that the GL theory only depends on a single dimensionless temperature independent coupling  $\kappa = H_{c2}(T) / (\sqrt{2}H_c(T)) = (mc/\hbar q)\sqrt{\beta/(2\pi)}$ , the upper critical field being  $H_{c2}(T) = \Phi_0 / 2\pi\xi(T)^2$ . Hereafter we use the dimensionless quantities:  $\vec{x} \rightarrow \vec{x}/\xi$  ( $\vec{\nabla} \rightarrow \xi\vec{\nabla}$ ),  $\vec{A} \rightarrow (2\pi\xi/\Phi_0)\vec{A}$  ( $\vec{D} \rightarrow (\hbar/\xi)\vec{D}$ ,  $\vec{D} \rightarrow \vec{\nabla}/i - \vec{A}$ ,  $\vec{h} \rightarrow \vec{h}/H_{c2}$  ( $\vec{H} \rightarrow \vec{H}/H_{c2}$ ),  $F \rightarrow F/(H_c^2/4\pi)$  ( $G \rightarrow G/(H_c^2/4\pi)$ ). For instance, the thermodynamic relation  $\vec{H} = 4\pi\partial F/\partial\vec{B}$  becomes in re-

duced units  $\vec{H} = (1/2\kappa^2)\partial F/\partial\vec{B}$ . Consider the special case of symmetry along the third direction defined by  $H$  and decompose the kinetic energy using the key condition to obtain that the free energy is  $F = \langle |D_+\psi|^2 + h_3|\psi|^2 - |\psi|^2 + |\psi|^4/2 + \kappa^2 h_3^2 \rangle$ . A straightforward rearrangement of the terms leads to  $F = -1/2 + B + F_0$ , where  $F_0 = \langle |D_+\psi|^2 \rangle + \langle (h_3 + |\psi|^2 - 1)^2 \rangle / 2 + (\kappa^2 - 1/2)\langle h_3^2 \rangle$ . For  $\kappa = 1/\sqrt{2}$   $F_0$  is positive and hold the first order differential equations,  $D_+\psi = 0$  and  $h_3 + |\psi|^2 - 1 = 0$ , at the minimum since  $F_0 = 0$ . Consequently, at this particular coupling, vortices do not interact as the free energy is additive in their number,  $N$ , and  $F = -1/2 + B$ , where the magnetic induction is  $B = N\Phi_0/S$ .

The 2COP GL theory which has a Bogomolny symmetry is,

$$F = \left\langle \frac{|\vec{D}\Psi|^2}{2m} - \vec{\alpha}(T) \cdot \Psi^\dagger \vec{\sigma} \Psi + \frac{\beta}{2} (\Psi^\dagger \Psi)^2 + \frac{\vec{h}^2}{8\pi} \right\rangle. \quad (17)$$

The out of plane coupling,  $\alpha_3$ , is associated to  $T_c$ , whereas the in plane ones,  $\alpha_1$  and  $\alpha_2$ , are related to  $\theta$  transitions. The simplest of all situations,  $\alpha_1 = \alpha_2 = 0$ , brings some insight into the problem since the remaining quadratic coupling becomes  $-\alpha_3(T)(|\psi_1|^2 - |\psi_2|^2)$ . In this case  $\psi_1$  admits a non-trivial solution below  $T_c$  ( $\alpha_3(T) > 0$ ), whereas  $\psi_2$  does not, and must vanish. However above  $T_c$  ( $\alpha_3(T) < 0$ ) a non-trivial solution for  $\psi_2$  must be prevented and this is done by assuming that  $\alpha_3$  vanishes above  $T_c$ . We get further insight into the properties of this theory by doing a  $SU(2)$  rotation  $\Psi_R = U(T)\Psi$ , such that  $\vec{\alpha}(T) \cdot U(T)\vec{\sigma}U(T)^\dagger = |\vec{\alpha}(T)|\sigma_3$ . Under this transformation the free energy becomes  $F = \langle |\vec{D}\Psi_R|^2 / 2m - |\vec{\alpha}(T)|\Psi_R^\dagger \sigma_3 \Psi_R + \beta(\Psi_R^\dagger \Psi_R)^2 / 2 + \vec{h}^2 / 8\pi \rangle$ . According to the previous argument we conclude that no stable second component is possible, namely,  $\psi_{R2} = 0$  turning this theory essentially into a 1COP GL theory for  $\psi_{R1}$ . Notice however an important difference,  $\psi_{R1}$  does not vanish at  $T_c$  because of the presence of  $\alpha_1$  and  $\alpha_2$ , assuming that these parameter only vanish above  $T_c$ . The previously defined quantities  $H_c(T)$ ,  $\xi(T)$ , and  $H_{c2}(T)$  depend on  $|\vec{\alpha}(T)|$  instead and this leads to the same dimensionless  $\kappa$  of the 1COP case. For instance the one-dimensional surface energy barrier for the 2COP theory of Eq.(17) is found to be  $\vec{\alpha}$  independent and only  $\kappa$  dependent, exactly like for the 1COP case, because the above rotation is possible. However for the three-dimensional problem we demand from the theory that the ground state condition of Eq.(8) applies to  $\Psi$  and not to  $\Psi_R$ . This will halt the rotation preventing the theory to become a one component free energy, but keeping its single scale properties. Reduced units are defined as before, and the free energy becomes,

$$F = \left\langle |\vec{D}\Psi|^2 - \frac{\vec{\alpha}}{|\vec{\alpha}|} \cdot \Psi^\dagger \vec{\sigma} \Psi + \frac{1}{2} (\Psi^\dagger \Psi)^2 + \kappa^2 \vec{h}^2 \right\rangle, \quad (18)$$

For zero magnetic field and constant order parameter the minimum of the 2COP GL potential gives the values for



the two physically relevant order parameters. Using that  $(\Psi^\dagger \Psi)^2 = (\Psi^\dagger \vec{\sigma} \Psi)^2$ , the GL potential becomes,  $F_P = -1/2 + (\Psi^\dagger \vec{\sigma} \Psi - \vec{\alpha}/|\vec{\alpha}|)^2/2$ , whose minimum is reached at  $|\psi_1|^2 = (|\vec{\alpha}| + \alpha_3)/2|\vec{\alpha}|$ ,  $|\psi_2|^2 = (|\vec{\alpha}| - \alpha_3)/2|\vec{\alpha}|$  and  $\tan \theta = -\alpha_2/\alpha_1$ . Notice that it is always true that  $|\psi_1| \geq |\psi_2|$ , and that above  $T_c$  they become equal, assuming that  $\alpha_1$  and  $\alpha_2$  exist above the transition. This restores a U(1) invariance to the theory. A change of sign in  $\alpha_1(T)$  or  $\alpha_2(T)$  changes the phase  $\theta$  but not the minimum of  $|\psi_1|$  and  $|\psi_2|$ . The transformation  $\alpha_1 \rightarrow -\alpha_1$  and  $\alpha_2 \rightarrow -\alpha_2$  is a symmetry of the minimum, equivalent to a  $\theta \rightarrow \theta + \pi$  rotation. This theory has a Bogomolny solution for  $\kappa = 1/\sqrt{2}$ , given by  $\vec{\sigma} \cdot \vec{D}\Psi = 0$  and  $\hbar + \Psi^\dagger \vec{\sigma} \Psi - \vec{\alpha}/|\vec{\alpha}| = 0$ , as the free energy can be expressed by,

$$F = \left\langle |\vec{\sigma} \cdot \vec{D}\Psi|^2 + [\vec{h} + (\Psi^\dagger \vec{\sigma} \Psi - \vec{\alpha}(T)/|\vec{\alpha}(T)|)]^2/2 \right\rangle + (\kappa^2 - 1/2) \langle \vec{h}^2 \rangle + (\vec{\alpha}(T)/|\vec{\alpha}(T)|) \cdot \vec{B}/2 - 1/2. \quad (19)$$

Thus the 2COP free energy studied here is distinct from all other free energies studied so far [8–12] because the ground state condition holds in two distinct applied field  $H$  and  $\kappa$  regimes, showing that it is a unique generalization of the 1COP GL theory:  $\kappa > 1/\sqrt{2}$  and  $0.5H_{c2} \leq H \leq H_{c2}$  (Abrikosov), and also  $\kappa = 1/\sqrt{2}$  throughout the whole  $H$  regime (Bogomolny).

In conclusion, we have found here a textured ground state for the two-component order parameter whose intrinsic transverse magnetic moment averages to zero along the superconducting layer because of  $\pi$  phase difference between distinct regions. This ground state solves one of the Bogomolny equations of a free energy obtained here. The ground state condition lives in a more fundamental level than that of the free energy expansion.

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